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Some comments on $\bar{n}p$-annihilation branching ratios into $\pi\pi$-, $\bar{K}K$- and $\pi\eta$-channels

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Abstract

We give some remarks on the $\bar{n}p$-partial branching ratios in flight at low momenta of antineutron, measured by OBELIX collaboration. The comparison is made to the known branching ratios from the $p\bar{p}$-atomic states. The branching ratio for the reaction $\bar{n}p \to \pi^+\pi^0$ is found to be suppressed in comparison to what follows from the $p\bar{p}$-data. It is also shown, that there is no so called dynamic $I = 0$–amplitude suppression for the process $N\bar{N} \to K\bar{K}$.

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1 Some useful definitions

Let us consider first the $N\bar{N}$-system. By definition $\mid I, I_3 >$ is the isospin wave function of the $N\bar{N}$-system with isospin I and its projection I_3. Using notations of ref. [1], we write the following relations between the physical states $\mid N\bar{N} >$ and states of definite isospin $\mid I, I_3 >$:

$$\mid p\bar{p} >= \frac{1}{\sqrt{2}}[\mid 1,0> - \mid 0,0>], \quad \mid n\bar{n} >= \frac{1}{\sqrt{2}}[\mid 1,0> + \mid 0,0>]. \tag{1}$$

On the contrary in terms of physical states the wave function $\mid I, I_3 >$ looks for isosinglet state as

$$\mid 0,0 >= -\frac{1}{\sqrt{2}}[\mid p\bar{p} > + \mid n\bar{n} >], \tag{2}$$

and for isotriplet as

$$\mid 1,-1 >=\mid n\bar{p} >, \mid 1,0 >= \frac{1}{\sqrt{2}}[\mid p\bar{p} > - \mid n\bar{n} >], \mid 1,1 >=\mid \bar{n}p > . \tag{3}$$

Each wave function is normalized as:

$$< N\bar{N} \mid N\bar{N} >= 1, \quad < I, I_3 \mid I, I_3 >= 1.$$

Let us also define wave function for the hadron final state $\mid a >$ with definite isospin I: $\mid a >_I$. We shall use the notations $\hat{V}_a^I$ for transition operator from initial $\mid I, I_3 >_{N\bar{N}}$-state to $\mid a >_I$ and

$$V_a^I =_I< a \mid \hat{V}_a^I \mid I, I_3 >_{N\bar{N}}, \tag{4}$$

is martix element for this operator. It doesn't depend on I_3. Evidently that

$$\hat{V}_a^I \mid J, J_3 >_{N\bar{N}}= 0$$

in the case $I \neq J$.

2 Matrix elements for the transitions $N\bar{N} \to \pi\pi$ and $N\bar{N} \to K\bar{K}$.

Consider only the transitions to the final $\pi\pi$-states from the initial $N\bar{N}$ S-wave ($^3S_1$). In this case the $\pi\pi$-system is produced in $I = 1$ isospin state. So there is only one operator $\hat{V}_\pi^1$. The expansion of the $\mid \pi\pi >$-wave function in terms of the states with definite isospin has the form:

$$\mid \pi^+\pi^- >= \frac{1}{\sqrt{3}} \mid 0,0 > +\frac{1}{\sqrt{2}} \mid 1,0 > +\frac{1}{\sqrt{6}} \mid 2,0 >, \tag{5}$$

$$\mid \pi^+\pi^0 >= \frac{1}{\sqrt{2}} \mid 1,1 > -\frac{1}{\sqrt{2}} \mid 2,1 > .$$

Thus using definitions (1), (3) and (4), we get

$$< \pi^+\pi^0 \mid \hat{V}_\pi^1 \mid \bar{n}p >= \frac{1}{\sqrt{2}} V_\pi^1, \tag{6}$$

$$< \pi^+\pi^- \mid \hat{V}_\pi^1 \mid p\bar{p} >= \frac{1}{2} V_\pi^1.$$

It means, that the processes $p\bar{p} \to \pi^+\pi^-$ is to be at least by factor two suppressed in comparison to $\bar{n}p \to \pi^+\pi^0$.

Let us now consider the transitions into $K\bar{K}$-final states. Isospin wave functions for $K\bar{K}$-states have the following form:

$$\mid K^+K^- >= \frac{1}{\sqrt{2}} [\mid 1,0 > - \mid 0,0 >], \tag{7}$$

$$\mid K^0\bar{K}^0 >= -\frac{1}{\sqrt{2}} [\mid 1,0 > + \mid 0,0 >],$$

$$\mid K^+\bar{K}^0 >= \mid 1,1 >, \qquad \mid K^0K^- >= - \mid 1,-1 > . \tag{8}$$

In this case $\mid K\bar{K} >$ final state is indeed a mixture of both $I = 0$ and $I = 1$ isospin states ($I_3 = 0$). Hence both operators $\hat{V}_K^1$ and $\hat{V}_K^0$ give contribution to this reaction, and

$$_K < 1, I_3 \mid \hat{V}_K^1 \mid 1, I_3 >_{N\bar{N}} = V_K^1, \qquad _K < 0,0 \mid \hat{V}_K^0 \mid 0,0 >_{N\bar{N}} = V_K^0.$$

In terms of V_K^1 and V_K^0 we may calculate matrix elements between the physical states:

$$< K^+K^- \mid \hat{V}_K \mid p\bar{p} >= \frac{V_K^0 + V_K^1}{2}, \qquad < K^0\bar{K}^0 \mid \hat{V}_K \mid p\bar{p} >= \frac{V_K^0 - V_K^1}{2}, \tag{9}$$

$$< K^+\bar{K}^0 \mid \hat{V}_K \mid \bar{n}p >= V_K^1. \tag{10}$$

The matrix elements (9),(10) are related to the corresponding partial cross-sections:

$$\sigma = 4\pi \frac{q}{k} \mid < f \mid V \mid i > \mid^2,$$

where q and k are final and initial c.m. momenta. We get the agreement for the expression (10) with what is given in the ref.[1], but expressions (9) differ from that of ref. [1]. Namely, redefining the operators according to equation (32) of ref. [1], we get:

$$\sigma(p\bar{p} \to K^+K^-) + \sigma(p\bar{p} \to K^0\bar{K}^0) = \mid A_0 \mid^2 + \mid A_1 \mid^2 \tag{11}$$

and

$$\sigma(\bar{n}p \to K^+\bar{K}^0) = 2 \mid A_1 \mid^2 . \tag{12}$$

Notice, that factor 2 in the right-hand side of the equation (12) is not present in equation (35) of the paper [1]. Historically this factor was also lost in the papers [2, 3], and this error was

reproduced later in some review papers, see, e.g. [4, 5]. That is why the conclusion of the papers [1, 2, 3] on $I = 0$-amplitude suppression seems to be incorrect and is to be revised. We shall discuss this problem in Section 4.

3 Some relations between branching ratios in $p\bar{p}$- and $\bar{n}p$-annihilation processes

Let us first consider the $\pi\pi$-case. By definition of the branching ratio we have:

$$Br_{\pi^+\pi^0}(\bar{n}p) = \frac{\sigma(\bar{n}p \to \pi^+\pi^-)}{\sigma(\bar{n}p \to all)}$$

and similar expression for the $p\bar{p}$-case. So the ratio of branching ratios is:

$$\frac{Br_{\pi^+\pi^0}(\bar{n}p)}{Br_{\pi^+\pi^-}(\bar{p}p)} = \frac{\sigma(\bar{n}p \to \pi^+\pi^0)}{\sigma(\bar{n}p \to all)} : \frac{\sigma(\bar{p}p \to \pi^+\pi^-)}{\sigma(\bar{p}p \to all)}. \tag{13}$$

Notice, that at low energies, if only S-wave contribute, we have:

$$\sigma(p\bar{n} \to \pi^+\pi^0) = 4\pi\frac{3}{4} \,|< \pi\pi \mid \hat{V}_\pi^1 \mid p\bar{n} >|^2 \,\frac{q}{k} \tag{14}$$

and

$$\sigma(p\bar{p} \to \pi^+\pi^-) = 4\pi\frac{3}{4}C^2(k) \,|< \pi\pi \mid \hat{V}_\pi^1 \mid p\bar{p} >|^2 \,\frac{q}{k}. \tag{15}$$

Here $C^2(k)$ is the Gamov factor,

$$C^2(k) = \frac{2\pi}{ka_B}/[1 - \exp(-\frac{2\pi}{ka_B})],$$

and $a_B = 57.6fm$ is the $p\bar{p}$-Bohr radius. Taking into account (13)-(15), we get:

$$\frac{Br_{\pi^+\pi^0}(\bar{n}p)}{Br_{\pi^+\pi^-}(\bar{p}p)} = \frac{|< \pi^+\pi^0 \mid \hat{V}_\pi^1 \mid \bar{n}p >|^2}{|< \pi^+\pi^- \mid \hat{V}_\pi^1 \mid \bar{p}p >|^2} \frac{[\beta C^{-2}(k)\sigma^{ann}(p\bar{p} \to all)]}{[\beta\sigma^{ann}(\bar{n}p \to all)]} \approx 2R, \tag{16}$$

where R is now a well defined and finite quantity:

$$R = \frac{\lim_{k\to 0}[\beta C^{-2}(k)\sigma^{ann}(p\bar{p})]}{\lim_{k\to 0}[\beta\sigma^{ann}(\bar{n}p)]}. \tag{17}$$

From the experimental data of refs. [6] and [8] we get the value of R at low momenta of incident antiproton ($p_{lab} = 50 - 70MeV/c$):

$$R = \frac{32 \pm 2}{25.3 \pm 1.0} \approx 1.26 \pm 0.10. \tag{18}$$

Notice, that this value coincides with what follows from the experimental data on annihilation of antiprotons off deuteron [7]. So we conclude, that the data [8] on total annihilation

$\bar{n}p$-cross section are in agreement with the results of quite independent experiment for the annihilation of antiproton on deuteron [7]. One may find the more detailed discussion of value R extracted from the different data on deuteron and some heavier nuclei in the review paper [9].

A case of kaons looks very similar. Using eqs. (9)-(10) as well as the definition of the ratio R (17), one gets the following relation between branchings for the reactions $p\bar{p} \to K^+K^-$, $p\bar{p} \to K^0\bar{K}^0$ and $\bar{n}p \to K^+\bar{K}^0$:

$$\frac{\mid V_K^1 \mid^2 + \mid V_K^0 \mid^2}{2 \mid V_K^1 \mid^2} = R\frac{Br(p\bar{p} \to K^+K^-) + Br(p\bar{p} \to K^0\bar{K}^0)}{Br(\bar{n}p \to K^+\bar{K}^0)}. \tag{19}$$

4 The analysis of the experimental situation

In Ref. [8] the branching ratio for the reaction $\bar{n}p \to \pi^+\pi^0$ in the momentum interval 50-150 MeV/c (S-wave) was found to be equal:

$$Br(\bar{n}p \to \pi^+\pi^0) = (2.3 \pm 0.4)10^{-3}. \tag{20}$$

This value is to be compared with what follows from the $(p\bar{p})$-atomic experiment for the reaction $p\bar{p} \to \pi^+\pi^-$. The separation of the S- and P-wave contribution to last reaction was provided in the Refs. [10, 11]. So we get for the branching ratio into $\pi^+\pi^-$-channel from atomic S-state:

a) $(2.37 \pm 0.23)10^{-3}$ [10]; b) $(2.04 \pm 0.17)10^{-3}$ [11].

Substituting these numbers into eq.(16), we get the evident contradiction. It means, that something is wrong with the branchings. If one believes in the experimental branchings for both $\bar{n}p$- and $\bar{p}p$-channels, the only possible way to solve the problem is to suggest, that the $p\bar{p}$-atomic wave function at small distances has an abnormal admixture of the $\bar{n}n$-component. We shall discuss this hypothesis in the next Section.

Now let us discuss a case of kaons. The only information on branching ratio $N\bar{N} \to K\bar{K}$ for isospin $I = 1$ channel for long time was available from the old data for absorption of antiproton on deuteron [12],

$$Br(\bar{p}n \to K^0K^-) = (1.47 \pm 0.21)10^{-3}.$$

Nowadays the OBELIX collaboration gives [1] (S-wave):

$$Br(\bar{n}p \to K^+K_S) = (0.92 \pm 0.23)10^{-3}.$$

It means, that the branching into $K^+\bar{K}^0$ is:

$$Br(\bar{n}p \to K^+\bar{K}^0) = 2Br(\bar{n}p \to K^+K_S) = (1.84 \pm 0.46)10^{-3}.$$

It is seen, that this last number for branching does not contradict the old data by Bettini et al. [12].

At the same time from the ASTERIX experiments [3,13] we have:

$$Br(p\bar{p} \to K^+K^-) = (1.08 \pm 0.05)10^{-3},$$

$$Br(p\bar{p} \to K^0\bar{K}^0) = (0.83 \pm 0.05)10^{-3}.$$

Using these values and taking into account equation (19), we get

$$\mid V_K^0 \mid \approx 1.3 \mid V_K^1 \mid . \tag{21}$$

So we conclude, that there is no evidence for any suppression of $I = 0$–amplitude for the reaction $N\bar{N} \to K\bar{K}$ in the S-wave. The dynamic selection rule for this process, declared in the Refs.[1-5] is the consequence of incorrect formulae for branchings used in refs.[1,2].

Let us also discuss a case of $\pi\eta$–channel. From the experiment [8] it follows, that in the momentum interval 150-250 MeV/c (P-wave)

$$Br(\bar{n}p \to \pi^+\eta) = (0.99 \pm 0.22)10^{-3}.$$

At the same time from the paper [10] we have:

$$Br(p\bar{p} \to \pi^0\eta) = (7.7 \pm 1.13)10^{-4}.$$

So again we come to the conclusion that the ratio

$$\frac{Br(\bar{n}p \to \pi^+\eta)}{Br(\bar{p}p \to \pi^0\eta)}$$

is significantly less than $2R$ (see eq.(16)).

5 Possible solution of the problem for the $N\bar{N} \to \pi\pi$ branchings

In line with the papers [1,14,15] let us suppose, that the wave function for $p\bar{p}$–atom at small distances is a superposition of $\mid p\bar{p} >$ and $\mid n\bar{n} >$ configurations, i.e.:

$$\mid \psi_{at} >= \frac{1}{\sqrt{1+\epsilon^2}}[\mid p\bar{p} > +\epsilon \mid n\bar{n} >]. \tag{22}$$

In terms of the states of definite isospin it means, that

$$\mid \psi_{at} >= \frac{1}{\sqrt{2(1+\epsilon^2)}}[(1-\epsilon) \mid 1,0 > -(1+\epsilon) \mid 0,0 >]. \tag{23}$$

So it follows immediately, that:

$$\frac{Br(\psi_{at} \to \pi^+\pi^-)}{Br(\bar{n}p \to \pi^+\pi^0)} = \frac{(1-\epsilon)^2}{2(1+\epsilon^2)R} \tag{24}$$

A case $\epsilon = 0$ corresponds to the usual suggestion of the absence of the $n\bar{n}$-component in the $p\bar{p}$–atom. In the limit $\epsilon = -1$ the atomic state is that of definite isospin $I = 1$. Substituting the experimental numbers for the $\pi\pi$-branchings (see Section 4), we conclude, that it is possible to fit the parameter ϵ so the equation (23) is justified. For example, taking $\mathrm{Br}(p\bar{p} \to \pi^+\pi^-) = 1.87$ (lower limit) and $\mathrm{Br}(\bar{n}p \to \pi^+\pi^0) = 2.7$ (upper limit), we get $\epsilon = -2.24$, that corresponds to the value of mixing angle $\cos\alpha = 1/\sqrt{1+\epsilon^2}$; $\alpha \approx 66°$. It means, that the admixture of the $\bar{n}n$–component should be large to fit the experimental data.

6 Conclusion

a) The data on the $\bar{n}p$–total annihilation cross section, presented by OBELIX Collaboration [8], are in agreement with the data on the value of the ratio R, determined from the absorption of antiprotons on deuteron (see [7] and references in [9]).

b) The branching ratios for the reactions $\bar{n}p \to \pi^+\pi^0$ and $\bar{n}p \to \pi^+\eta$ at low energies [8] seem to be too large in comparison to what follows from the analysis of the known branching ratios for the $p\bar{p}$–atom.

c) The branching for the reaction $\bar{n}p \to K^+K_S$ is in agreement to the known branching for the reaction $\bar{p}n \to K^0K^-$ from the deuteron data [12]. There is no suppression for the $I = 0$ $N\bar{N} \to K\bar{K}$–reaction amplitude in S-wave (no specific dynamic selection rule).

d) Some admixture of the $\mid n\bar{n} >$-component in the $p\bar{p}$-atomic wave function may help in solving the problems with the branching into two pions and $\pi\eta$. However to solve this problem, the admixture should be large enough.

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